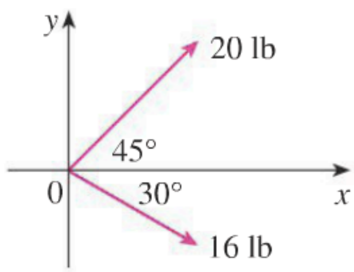
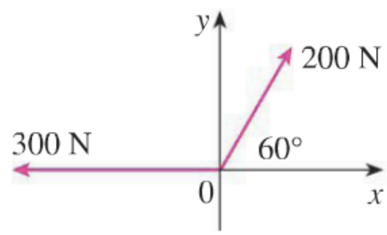
Additional Vector Application Problems

32–33 Find the magnitude of the resultant force and the angle it makes with the positive x-axis.

32.



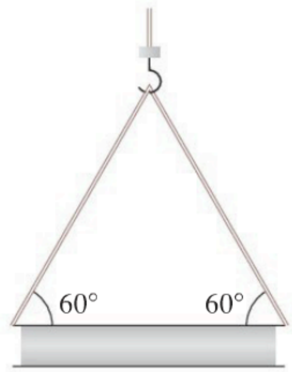
33.



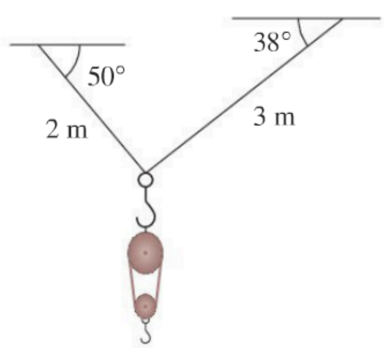
34. The magnitude of a velocity vector is called *speed*. Suppose that a wind is blowing *from* the direction N45°W at a speed of 50 km/h. (This means that the direction from which the wind blows is 45° west of the northerly direction.) A pilot is steering a plane in the direction N60°E at an airspeed (speed in still air) of 250 km/h. The *true course,* or *track,* of the plane is the direction of the resultant of the velocity vectors of the plane and the wind. The *ground speed* of the plane is the magnitude of the resultant. Find the true course and the ground speed of the plane.

35. A woman walks due west on the deck of a ship at 3 mi/h. The ship is moving north at a speed of 22 mi/h. Find the speed and direction of the woman relative to the surface of the water.

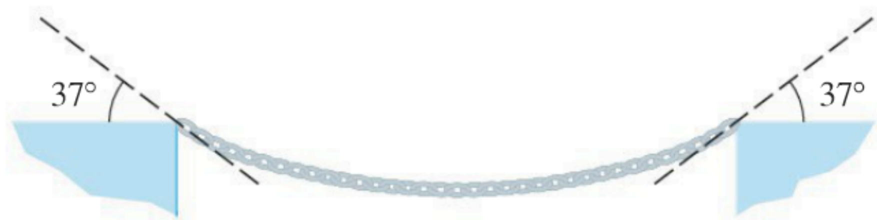
36. A crane suspends a 500-lb steel beam horizontally by support cables (with negligible weight) attached from a hook to each end of the beam. The support cables each make an angle of 60° with the beam. Find the tension vector in each support cable and the magnitude of each tension.



37. A block-and-tackle pulley hoist is suspended in a warehouse by ropes of lengths 2 m and 3 m. The hoist weighs 350 N. The ropes, fastened at different heights, make angles of 50° and 38° with the horizontal. Find the tension in each rope and the magnitude of each tension.



38. The tension **T** at each end of a chain has magnitude 25 N (see the figure). What is the weight of the chain?



39. A boatman wants to cross a canal that is 3 km wide and wants to land at a point 2 km upstream from his starting point. The current in the canal flows at 3.5 km/h and the speed of his boat is 13 km/h.

(a) In what direction should he steer?

(b) How long will the trip take?

40. Three forces act on an object. Two of the forces are at an angle of 100° to each other and have magnitudes 25 N and 12 N. The third is perpendicular to the plane of these two forces and has magnitude 4 N. Calculate the magnitude of the force that would exactly counterbalance these three forces.

32. The given force vectors can be expressed in terms of their horizontal and vertical components as $20\cos 45°\,\mathbf{i} + 20\sin 45°\,\mathbf{j} = 10\sqrt{2}\,\mathbf{i} + 10\sqrt{2}\,\mathbf{j}$ and $16\cos 30°\,\mathbf{i} - 16\sin 30°\,\mathbf{j} = 8\sqrt{3}\,\mathbf{i} - 8\,\mathbf{j}$. The resultant force $\mathbf{F}$ is the sum of these two vectors: $\mathbf{F} = \left(10\sqrt{2} + 8\sqrt{3}\right)\mathbf{i} + \left(10\sqrt{2} - 8\right)\mathbf{j} \approx 28.00\,\mathbf{i} + 6.14\,\mathbf{j}$. Then we have $|\mathbf{F}| \approx \sqrt{(28.00)^2 + (6.14)^2} \approx 28.7$ lb and, letting θ be the angle $\mathbf{F}$ makes with the positive x-axis,

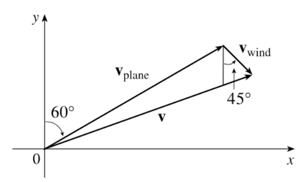
$$\tan\theta = \frac{10\sqrt{2} - 8}{10\sqrt{2} + 8\sqrt{3}} \quad\Rightarrow\quad \theta = \tan^{-1}\left(\frac{10\sqrt{2} - 8}{10\sqrt{2} + 8\sqrt{3}}\right) \approx 12.4°.$$

33. The given force vectors can be expressed in terms of their horizontal and vertical components as $-300\,\mathbf{i}$ and $200\cos 60°\,\mathbf{i} + 200\sin 60°\,\mathbf{j} = 200\left(\frac{1}{2}\right)\,\mathbf{i} + 200\left(\frac{\sqrt{3}}{2}\right)\,\mathbf{j} = 100\,\mathbf{i} + 100\sqrt{3}\,\mathbf{j}$. The resultant force $\mathbf{F}$ is the sum of these two vectors: $\mathbf{F} = (-300 + 100)\,\mathbf{i} + \left(0 + 100\sqrt{3}\right)\mathbf{j} = -200\,\mathbf{i} + 100\sqrt{3}\,\mathbf{j}$. Then we have $|\mathbf{F}| \approx \sqrt{(-200)^2 + \left(100\sqrt{3}\right)^2} = \sqrt{70{,}000} = 100\sqrt{7} \approx 264.6$ N. Let θ be the angle $\mathbf{F}$ makes with the positive x-axis. Then $\tan\theta = \dfrac{100\sqrt{3}}{-200} = -\dfrac{\sqrt{3}}{2}$ and the terminal point of $\mathbf{F}$ lies in the second quadrant, so

$$\theta = \tan^{-1}\left(-\frac{\sqrt{3}}{2}\right) + 180° \approx -40.9° + 180° = 139.1°.$$

34. Set up the coordinate axes so that north is the positive y-direction, and east is the positive x-direction. The wind is blowing at 50 km/h from the direction N$45°$W, so that its velocity vector is 50 km/h S$45°$E, which can be written as $\mathbf{v}_{\text{wind}} = 50(\cos 45°\mathbf{i} - \sin 45°\mathbf{j})$. With respect to the still air, the velocity vector of the plane is 250 km/h N$\,60°$E, or equivalently $\mathbf{v}_{\text{plane}} = 250(\cos 30°\mathbf{i} + \sin 30°\mathbf{j})$. The velocity of the plane relative to the ground is

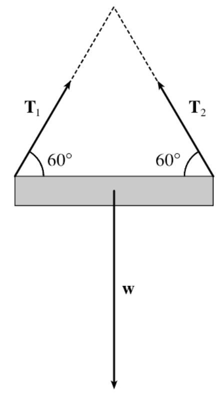
$$\begin{aligned}
\mathbf{v} &= \mathbf{v}_{\text{plane}} + \mathbf{v}_{\text{wind}} \\
&= (250\cos 30° + 50\cos 45°)\,\mathbf{i} + (250\sin 30° - 50\sin 45°)\,\mathbf{j} \\
&= \left(125\sqrt{3} + 25\sqrt{2}\right)\mathbf{i} + \left(125 - 25\sqrt{2}\right)\mathbf{j} \\
&\approx 251.9\,\mathbf{i} + 89.6\,\mathbf{j}
\end{aligned}$$



(See the figure.) The ground speed is $|\mathbf{v}| \approx \sqrt{(251.9)^2 + (89.6)^2} \approx 267$ km/h. The angle the velocity vector makes with the x-axis is $\theta \approx \tan^{-1}\left(\frac{89.6}{251.9}\right) \approx 20°$. Therefore, the true course of the plane is about N$(90 - 20)°$E $=$ N$\,70°$E.

35. With respect to the water's surface, the woman's velocity is the vector sum of the velocity of the ship with respect to the water, and the woman's velocity with respect to the ship. If we let north be the positive y-direction, then $\mathbf{v} = \langle 0, 22\rangle + \langle -3, 0\rangle = \langle -3, 22\rangle$. The woman's speed is $|\mathbf{v}| = \sqrt{9 + 484} \approx 22.2 \text{ mi/h}$. The vector $\mathbf{v}$ makes an angle θ with the east, where $\theta = \tan^{-1}\left(\frac{22}{-3}\right) \approx 98^\circ$. Therefore, the woman's direction is about $\text{N}(98 - 90)^\circ\text{W} = \text{N}8^\circ\text{W}$.

36. Let $\mathbf{T}_1$ and $\mathbf{T}_2$ be the tension vectors corresponding to the support cables as shown in the figure. In terms of vertical and horizontal components,



$$\mathbf{T}_1 = |\mathbf{T}_1|\cos 60^\circ\mathbf{i} + |\mathbf{T}_1|\sin 60^\circ\mathbf{j} = \frac{1}{2}\,|\mathbf{T}_1|\;\mathbf{i} + \frac{\sqrt{3}}{2}\,|\mathbf{T}_1|\;\mathbf{j}$$

$$\mathbf{T}_2 = -\,|\mathbf{T}_2|\cos 60^\circ\mathbf{i} + |\mathbf{T}_2|\sin 60^\circ\mathbf{j} = -\frac{1}{2}\,|\mathbf{T}_2|\;\mathbf{i} + \frac{\sqrt{3}}{2}\,|\mathbf{T}_2|\;\mathbf{j}$$

The resultant of these tensions, $\mathbf{T}_1 + \mathbf{T}_2$, counterbalances the weight $\mathbf{w} = -500\,\mathbf{j}$. So $\mathbf{T}_1 + \mathbf{T}_2 = -\mathbf{w} = 500\,\mathbf{j} \quad\Rightarrow$

$$\left(\frac{1}{2}\,|\mathbf{T}_1|\;\mathbf{i} + \frac{\sqrt{3}}{2}\,|\mathbf{T}_1|\;\mathbf{j}\right) + \left(-\frac{1}{2}\,|\mathbf{T}_2|\;\mathbf{i} + \frac{\sqrt{3}}{2}\,|\mathbf{T}_2|\;\mathbf{j}\right) = 500\,\mathbf{j}.$$

Equating x-components gives $\frac{1}{2}\,|\mathbf{T}_1|\;\mathbf{i} - \frac{1}{2}\,|\mathbf{T}_2|\;\mathbf{i} = 0$, so $|\mathbf{T}_1| = |\mathbf{T}_2|$ (as we would expect from the symmetry of the problem). Equating y-components, we have $\frac{\sqrt{3}}{2}\,|\mathbf{T}_1|\;\mathbf{j} + \frac{\sqrt{3}}{2}\,|\mathbf{T}_2|\;\mathbf{j} = \sqrt{3}\,|\mathbf{T}_1|\;\mathbf{j} = 500\,\mathbf{j} \quad\Rightarrow\quad |\mathbf{T}_1| = \frac{500}{\sqrt{3}}$. Thus the magnitude of each tension is $|\mathbf{T}_1| = |\mathbf{T}_2| = \frac{500}{\sqrt{3}} \approx 288.68 \text{ lb}$. The tension vectors are

$\mathbf{T}_1 = \frac{1}{2}\,|\mathbf{T}_1|\;\mathbf{i} + \frac{\sqrt{3}}{2}\,|\mathbf{T}_1|\;\mathbf{j} = \frac{250}{\sqrt{3}}\,\mathbf{i} + 250\,\mathbf{j} \approx 144.34\,\mathbf{i} + 250\,\mathbf{j}$ and $\mathbf{T}_2 = -\frac{250}{\sqrt{3}}\,\mathbf{i} + 250\,\mathbf{j} \approx -144.34\,\mathbf{i} + 250\,\mathbf{j}$.

37. Call the two tension vectors $\mathbf{T}_2$ and $\mathbf{T}_3$, corresponding to the ropes of length 2 m and 3 m. In terms of vertical and horizontal components,

$$\mathbf{T}_2 = -|\mathbf{T}_2|\cos 50°\mathbf{i} + |\mathbf{T}_2|\sin 50°\mathbf{j} \quad \textbf{(1)} \qquad \text{and} \qquad \mathbf{T}_3 = |\mathbf{T}_3|\cos 38°\mathbf{i} + |\mathbf{T}_3|\sin 38°\mathbf{j} \quad \textbf{(2)}$$

The resultant of these forces, $\mathbf{T}_2 + \mathbf{T}_3$, counterbalances the weight of the hoist (which is $-350\,\mathbf{j}$), so

$\mathbf{T}_2 + \mathbf{T}_3 = 350\,\mathbf{j} \quad \Rightarrow$

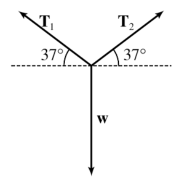
$(-|\mathbf{T}_2|\cos 50° + |\mathbf{T}_3|\cos 38°)\,\mathbf{i} + (|\mathbf{T}_2|\sin 50° + |\mathbf{T}_3|\sin 38°)\,\mathbf{j} = 350\,\mathbf{j}$. Equating components, we have

$-|\mathbf{T}_2|\cos 50° + |\mathbf{T}_3|\cos 38° = 0 \quad \Rightarrow \quad |\mathbf{T}_2| = |\mathbf{T}_3|\dfrac{\cos 38°}{\cos 50°}$ and

$|\mathbf{T}_2|\sin 50° + |\mathbf{T}_3|\sin 38° = 350$. Substituting the first equation into the second gives

$|\mathbf{T}_3|\dfrac{\cos 38°}{\cos 50°}\sin 50° + |\mathbf{T}_3|\sin 38° = 350 \quad \Rightarrow \quad |\mathbf{T}_3|\,(\cos 38°\tan 50° + \sin 38°) = 350$, so the magnitudes of the tensions are $|\mathbf{T}_3| = \dfrac{350}{\cos 38°\tan 50° + \sin 38°} \approx 225.11$ N and $|\mathbf{T}_2| = |\mathbf{T}_3|\dfrac{\cos 38°}{\cos 50°} \approx 275.97$ N. Finally, from **(1)** and **(2)**, the tension vectors are $\mathbf{T}_2 \approx -177.39\,\mathbf{i} + 211.41\,\mathbf{j}$ and $\mathbf{T}_3 \approx 177.39\,\mathbf{i} + 138.59\,\mathbf{j}$.

38. We can consider the weight of the chain to be concentrated at its midpoint. The forces acting on the chain then are the tension vectors $\mathbf{T}_1$, $\mathbf{T}_2$ in each end of the chain and the weight $\mathbf{w}$, as shown in the figure. We know $|\mathbf{T}_1| = |\mathbf{T}_2| = 25$ N so, in terms of vertical and horizontal components, we have

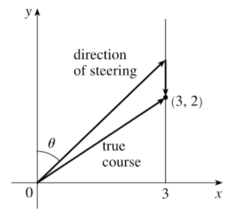


$$\mathbf{T}_1 = -25\cos 37°\mathbf{i} + 25\sin 37°\mathbf{j} \qquad \mathbf{T}_2 = 25\cos 37°\mathbf{i} + 25\sin 37°\mathbf{j}$$

The resultant vector $\mathbf{T}_1 + \mathbf{T}_2$ of the tensions counterbalances the weight $\mathbf{w}$, giving $\mathbf{T}_1 + \mathbf{T}_2 = -\mathbf{w}$. Since $\mathbf{w} = -|\mathbf{w}|\,\mathbf{j}$, we have $(-25\cos 37°\mathbf{i} + 25\sin 37°\mathbf{j}) + (25\cos 37°\mathbf{i} + 25\sin 37°\mathbf{j}) = |\mathbf{w}|\,\mathbf{j} \quad \Rightarrow \quad 50\sin 37°\mathbf{j} = |\mathbf{w}|\,\mathbf{j} \quad \Rightarrow$

$|\mathbf{w}| = 50\sin 37° \approx 30.1$. So the weight is 30.1 N, and since $w = mg$, the mass is $\frac{30.1}{9.8} \approx 3.07$ kg.

39. (a) Set up coordinate axes so that the boatman is at the origin, the canal is bordered by the y-axis and the line $x = 3$, and the current flows in the negative y-direction. The boatman wants to reach the point $(3, 2)$. Let θ be the angle, measured from the positive y-axis, in the direction he should steer. (See the figure.)



In still water, the boat has velocity $\mathbf{v}_b = \langle 13\sin\theta, 13\cos\theta\rangle$ and the velocity of the current is $\mathbf{v}_c\ \langle 0, -3.5\rangle$, so the true path of the boat is determined by the velocity vector $\mathbf{v} = \mathbf{v}_b + \mathbf{v}_c = \langle 13\sin\theta, 13\cos\theta - 3.5\rangle$. Let t be the time (in hours) after the boat departs; then the position of the boat at time t is given by $t\mathbf{v}$ and the boat crosses the canal when

$t\mathbf{v} = \langle 13\sin\theta, 13\cos\theta - 3.5\rangle\, t = \langle 3, 2\rangle$. Thus $13(\sin\theta)t = 3 \quad\Rightarrow\quad t = \dfrac{3}{13\sin\theta}$ and $(13\cos\theta - 3.5)\, t = 2$.

Substituting gives $(13\cos\theta - 3.5)\,\dfrac{3}{13\sin\theta} = 2 \quad\Rightarrow\quad 39\cos\theta - 10.5 = 26\sin\theta$ **(1)**. Squaring both sides, we have

$$1521\cos^2\theta - 819\cos\theta + 110.25 = 676\sin^2\theta = 676\left(1 - \cos^2\theta\right)$$

$$2197\cos^2\theta - 819\cos\theta - 565.75 = 0$$

The quadratic formula gives

$$\cos\theta = \frac{819 \pm \sqrt{(-819)^2 - 4(2197)(-565.75)}}{2(2197)}$$

$$= \frac{819 \pm \sqrt{5{,}642{,}572}}{4394} \approx 0.72699 \text{ or } -0.35421$$

The acute value for θ is approximately $\cos^{-1}(0.72699) \approx 43.4^\circ$. Thus the boatman should steer in the direction that is 43.4° from the bank, toward upstream.

Alternate solution: We could solve **(1)** graphically by plotting $y = 39\cos\theta - 10.5$ and $y = 26\sin\theta$ on a graphing device and finding the appoximate intersection point $(0.757, 17.85)$. Thus $\theta \approx 0.757$ radians or equivalently 43.4°.

(b) From part (a) we know the trip is completed when $t = \dfrac{3}{13\sin\theta}$. But $\theta \approx 43.4^\circ$, so the time required is approximately $\dfrac{3}{13\sin 43.4^\circ} \approx 0.336$ hours or 20.2 minutes.

40. Let $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ be the force vectors where $|\mathbf{v}_1| = 25$, $|\mathbf{v}_2| = 12$, and $|\mathbf{v}_3| = 4$. Set up coordinate axes so that the object is at the origin and $\mathbf{v}_1$, $\mathbf{v}_2$ lie in the xy-plane. We can position the vectors so that $\mathbf{v}_1 = 25\,\mathbf{i}$, $\mathbf{v}_2 = 12\cos 100^\circ\,\mathbf{i} + 12\sin 100^\circ\,\mathbf{j}$, and $\mathbf{v}_3 = 4\,\mathbf{k}$. The magnitude of a force that counterbalances the three given forces must match the magnitude of the resultant force. We have $\mathbf{v}_1 + \mathbf{v}_2 + \mathbf{v}_3 = (25 + 12\cos 100^\circ)\,\mathbf{i} + 12\sin 100^\circ\,\mathbf{j} + 4\,\mathbf{k}$, so the counterbalancing force must have magnitude $|\mathbf{v}_1 + \mathbf{v}_2 + \mathbf{v}_3| = \sqrt{(25 + 12\cos 100^\circ)^2 + (12\sin 100^\circ)^2 + 4^2} \approx 26.1$ N.